9470 – Mathematics II

General comments

This was an accessible paper, with up to half the marks on each question available to candidates of a suitable potential, The candidature represented the usual range of mathematical talents, with a goodly number of truly outstanding students, many more who were able to show insight and flair on some of the questions they attempted, and (sadly) a significant number of students for whom the experience was not to prove a particularly profitable one. Of the total entry of nearly 700, around 40% were awarded grade 1's (or better), while only around 20% received an unclassified grade.

Really able candidates generally produced solid attempts at six questions, while the weaker brethren were often to be found scratching around at bits and pieces of several questions, with little of substance being produced. In general, few candidates submitted serious attempts at more than six questions – a practice that is not to be encouraged, as it uses valuable examination time to little or no avail. It is, therefore, important for candidates to spend a few minutes at some stage of the examination deciding upon their optimal selection of questions to attempt.

As a rule, question 1 is intended to be accessible to all takers, with question 2 usually similarly constructed. In the event, at least one – and usually both – of these two questions were among candidates' chosen questions. Of the remaining selections, the majority of candidates supplied attempts at the questions in Section A (Pure Maths) only. There were relatively few attempts at the Applied Maths questions in Sections B & C, with Mechanics proving by far the more popular of the two options. Question 10, in particular, was relatively popular. Overall, there were remarkably few efforts submitted to the Statistics questions in Section C, although several of these were of exceptional quality.

On a more technical note, many solutions to those questions which were not already quite structured suffered a lamentable lack of clearly directed working. Large numbers of candidates would benefit considerably from the odd comment to indicate the direction that their working was taking. This was especially the case in questions 3, 5, 10 and 13, where it was often very difficult for examiners to decide what candidates were attempting to do, and where they had gone wrong, without any clear indication as to what they themselves thought they were doing.

Comments on individual questions

1 Almost all candidates attempted this question and most managed at least some measure of success; although the high level of algebra required to see matters through to a successful conclusion proved to be a decisive factor in whether attempts got much over half-marks. A minority of candidates worked with u_n and u_{n+r} (for the appropriate r's) and thereby made the algebra rather harder for themselves; whereas it had been intended that they should work with u_1 (with the given value of 2) and the appropriate u_r in order to determine periodicity. The other major problem arose when candidates worked backwards from (say) u_5 towards u_1, rather than forwards. This often generated nested sets of bracketed expressions of the form

$$u_5 = k - \cfrac{36}{k - \cfrac{36}{k - \cfrac{36}{k - \dots}}}$$

which only the hardiest were able to unravel successfully; while a forwards approach would have found each of u_2, u_3, … successively as much simpler (rational) terms.

Another common error arose when candidates failed to note that, if $k = 20$ gives a constant sequence, then, for a sequence of period 2, the answers "$k = 20$ and 0" can't both be correct. Similarly, for a sequence of period 4, the values 0 and 20 should appear as possible solutions when equating u_5 to u_1, but should be discounted. Whilst many candidates noted these points – and some shrewdly used their existence to help factorise the arising quartic equation in k – it is still clearly the case that a large proportion of A-level students, even the better ones, are happy to assume that any solution to an equation they end up having to solve is valid, irrespective of the context of the underlying problem or the logic of their work (viz. *necessary and/or sufficient conditions*).

Although only the most basic of arguments was required to establish that $u_n \geq 2$ at the beginning of part (ii), it was clear that most candidates were really not comfortable handling inequalities, and lacked practice in constructing reasonable mathematical arguments. Far too many failed to work generally at all, and simply showed that the first few terms were greater than or equal to 2, concluding with a waffle-y "etc., etc., etc." sort of argument. In the very last part, it was important to appreciate that a limit is approached when successive terms effectively become the same. No formal work beyond this simple idea was required, and the resulting quadratic gave two solutions, only one of which was greater than 2. Rather a lot of candidates were happy with this idea and rattled it through very quickly.

2 This question was the second most popular on the paper (in terms of the number of attempts) and really sorted out those who were comfortable with inequalities from those that weren't. Those who were generally scored very high marks on the question; even those who weren't generally managed several bits and pieces to get around half-marks on it.

Once again, there was an informal (possibly induction-type) proof required for the second bit of the question, although this was handled slightly more capably than the easier one in Q1, possibly because so many candidates seemed happier to effectively produce a formally inductive line of reasoning. Most candidates then picked up on the purpose of this bit in helping create a convergent GP to sum, which helped establish the next inequality for e.

The differentiation proved undemanding, and most candidates managed to realise that the minimum and maximum points referred to would be established by considering the sign of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = \frac{1}{2}$, 1 and $\frac{5}{4}$. Rather fewer were entirely happy to use the given bounds on e to help them do so, with many going off to lengthier (although often equally correct) workings-out. (In the final part, the use of $e < 3$ would have done the trick.) Those candidates who used approximations rather than inequalities were missing the point, as were those who tried to use $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ without actually knowing the exact values of x which they could use in it.

3 A lot of candidates made a faltering start to this question before moving on to pastures greener. This was usually occasioned by a realisation that life was going to be very tough here – which it was if they failed to appreciate that $\frac{1}{5+\sqrt{24}} = 5 - \sqrt{24}$. Those who saw this early on generally made their way to at least the first 8 marks. Although there are other

ways to go about the first part, the use of the binomial theorem, with the $\sqrt{24}$ -bits all cancelling out, establishes that the given expression is indeed an integer (without necessarily having to find out which). The three modest inequalities that followed were easily established with just a modicum of care. However, it was again the case that candidates' lack of comfort with inequalities once more prevented a convincing conclusion to (i) since most candidates resorted to approximation: showing that $N \approx 9601.9999$ is NOT the same as showing that, because N lies between … and … , it is actually equal to it (to four decimal places). Sadly, most candidates did not seem to understand such a difference in logical terms.

For part (ii), it was necessary only to mimic the work of part (i) but in a general setting. Most candidates attempting this question were happy to leave it at this point; of those who continued, many picked up two or three marks – only a handful actually polished it off properly.

4 Another difficult start again put most candidates off this question at the outset (if not before) and there were relatively few efforts at it. Most of these were pretty decent and scored well. The use of the initial result in (i) was straightforward, provided one is prepared to spot a decent substitution (such as $c = \cos x$). The formula books then helped bypass the integration required. In (ii), the given integral splits into the answer to (i) + a second integral, which must be considered separately. A simple linear substitution helped here, although quite a few candidates incorrectly assumed a result over the interval $(\pi, 2\pi)$ similar to the given one could just be assumed to hold. This was often the case in (iii) also, although fewer candidates tried such a move: the $\sin(2x)$ forcing them to consider more sensible approaches, such as (again) a linear substitution (after using the double angle formula for sine).

5 Despite the introduction of a non-standard function – often called the *floor* or the *INT* function – this was a popular question to attempt. As mentioned earlier, finding the areas required candidates to structure their working and, since there are several ways to break up the bits of the process, a teensy-weensy bit of explanation would have been greatly appreciated by the examiners. The easiest approach to the area in (i) is to work straightaway with the difference $(y_1 - y_2)$ which immediately gives a whole load of "unit triangles" to sum. Attempts varied from excellent-and-concise all the way down to scrambled-heap-of-integrations-and summations. Part (ii) was handled similarly, although it is strange to say that – despite the slightly greater degree of care needed with the various bits and pieces – there were slightly more correct answers arrived at here.

6 In hindsight, it might have been more generous to have included an "or otherwise" option to the very opening part of this question, as many candidates – particularly overseas ones – preferred an algebraic approach to obtaining the given result, rather than the vector one asked-for. It does, however, illustrate a pretty important examination point: namely, that if you don't actually answer the question that has been asked, you may not actually get any marks for your time and effort! These candidates reduced the given inequality to

$$(bx - ay)^2 + (cy - bz)^2 + (az - cx)^2 \geq 0,$$

and this represents some pretty decent mathematics. It is also very easy to deduce when equality holds in the result from this alternative statement. Such candidates were able to get the remaining sixteen marks on the question, however.

Part (i) didn't actually require candidates to use the given result to solve this quadratic equation, but those who did were guided towards the helpful notion of considering the equality case of the given result, which was intended to help them approach part (ii). [The question cites an example of a result widely known as the *Cauchy-Schwarz Inequality*.]

Overseas candidates apart, this was not a very popular question at all. Those who attempted it generally did quite well, and a surprisingly high proportion of them saw it through right to the end.

7 This proved to be a relatively popular choice of question, usually being pretty well-done, at least up to the point where trig. identities came into play, and often all the way through. It is suspected that the principal reasons for this were that the question had a fairly routine start, and then developed in a fairly straightforward A-level manner thereafter.

Most attempts established the opening result easily enough, and also managed to acquire Q's coordinates without much difficulty, and usually the equation of the line PQ also. A common shortfall at the next stage was not so much the introduction of the trig., which clearly put some candidates off, but rather the use of the trig. to show that the two lines were the same when these identities were used. A very surprising number of candidates seemed content to suggest that the two forms of the line were the same **on the basis of their gradients only**.

Those who got as far as the last part usually handled it very capably, showing that the two cases led to PQ being the vertical and horizontal tangents (respectively) to the ellipse.

8 Clearly vectors weren't a popular choice for candidates, as there were very few attempts made at this question. The first six marks, however, are gifts and almost all attemptees gained these. Thereafter, it is simply a case, with (admittedly) increasingly complicated looking position vectors coming into play, of equating **a**'s and **c**'s in pairs of lines to find out the position vector of the point where they intersect. Candidates' efforts tailed off fairly uniformly as the question progressed, and examiners cannot recall anyone actually getting to the end and finding **h** (the p.v. of H) correctly, although there were several attempts that gained all but the final two marks.

9 These leaning-ladder questions are actually pretty standard, and it was disappointing to see so few attempts made at this one. More disappointing still was the lack of a decent diagram from which candidates might have been able to extract some support for their working. Similar dismay was evoked by the widespread inability, on the part of almost all candidates, to be able to say what mechanical principles they were attempting to use at any stage of their working. Of the relatively small number of attempts seen, most suffered from at least one of these deficiencies. Consequently, although there were many partially or totally successful attempts at (i), the number of even half-decent attempts at (ii) were very few. The extra forces that needed to be considered in (ii) were either overlooked completely, or were missing from (i)'s diagram that candidates were trying to re-use.

The other painfully obvious shortfall here lay in candidates' dislike of using the *Friction Law* in its more general, inequality, statement rather than in the equality case given by limiting equilibrium. Such a shortfall was overlooked, even when it wasn't explained correctly (although it contributed substantially to problems in part (ii), when working was to be found). Those making a stab at (i) usually managed to make correct statements from resolving and taking moments, although arguments putting everything together and explaining why the ladder was stable were often less than entirely satisfactory.

10 The most popular of the three Mechanics questions, and generally the best done. Even so, marking was often made unnecessarily difficult by candidates' failure to explain what was going on and/or simplify their working at suitable stages in the proceedings. Setting up and finding the post-collision velocities of the various particles was relatively straightforward – although the algebra did prove too demanding for quite a few candidates – and most attempts correctly indicated the condition required to give a second collision between A and B. The number of unsuccessful attempts to solve the resulting quadratic was a surprise – most presumably faltering due to the lack of a unit x^2 term! – as was the number who preferred to use the quadratic formula rather than factorisation.

Problems generally arose here in part (ii), where a lack of explanation was a big problem. Those candidates who simply work out times and distances, without saying what they are supposed to be, do themselves no favours, as it is very difficult for the examiners to give credit to the working until a coherent strategy has emerged. Any error, no matter how small – and especially those made by candidates working "in their heads" – can render it almost impossible to spot such a strategy and reward it. On a more fundamental level, part (ii) should have opened up with the statement of the three relevant velocities, given in terms of u, using $k = 1$. Most efforts made mistakes because this simple task was left until much later on in the working, and some candidates even insisted on working with a general k throughout.

11 It was felt by examiners that this was the nicest (and easiest) of the three Mechanics questions, yet it drew very few serious attempts from the candidature. Most serious efforts coped very easily with the first two parts. Thereafter, it was often the case that maximising OA proved to be a greater difficulty than it should have done, despite the fact that the option to use calculus was available (although much less concise an approach than using a trigonometric one). There had been concerns that, for the final part, candidates might not grasp what was going on but, happily, this proved not to be the case and several candidates spotted the significance of having $f = g$ **and** described the resulting motion adequately.

12 This was the least popular of the Statistics questions, even amongst the relatively small number of attempts at any Section C questions. Of those seen, examiners can recall only two which got the answer of $\frac{3}{10}$ in (i). This was due to the almost total lack of appreciation that the result "1 wicket taken" required three probabilities.

The clear guidance towards the use of a Poisson distribution in (ii) and (iii) was, however, picked up by candidates. The calculation of the ensuing probabilities, either directly or via tables, was actually very straightforward, and candidates coped very easily when they ventured this far.

13 To be honest, this was more of a counting question than anything, at least to begin with, and several candidates picked up relatively large amounts of marks for very little working. Whilst several attacked (i) by multiplying and adding various probabilities, it was possibly most easily approached by looking at the 24 permutations of {1, 2, 3, 4} individually. Those candidates who adopted a *mix-'n'-match* approach without explanation often got themselves into a bit of a muddle, but still picked up several of the marks available here.

The example provided by (i) was intended to help direct candidates' thinking in (ii) as well as give them with a non-trivial case to use as a check. Of the attempts received, many explained things very poorly, even when they arrived at the correct expression. Sadly, rather too many seemed to deduce the (correct) answer on the basis of (i)'s example alone, and seemed unable to grasp that anything needed to be explained or justified.

14 This was a relatively popular choice of question, perhaps partially because it started off with a couple of bits of Pure Maths: namely, curve-sketching and integration. Strangely, though, very few sketches were fully correct, even when followed-through by "reciprocating" a correct sketch of $y = x \ln x$.

Further progress was going to be impossible without integrating $\frac{1}{x \ln x}$, and some attempts fell at this hurdle. Pleasingly, several candidates spotted the log. form immediately, while many others correctly used the substitution $u = \ln x$, or equivalent.

Thereafter, it was a routine statistical exercise in some respects. However, the log. work required to simplify matters in (i) proved beyond rather too many candidates – whereas it proved much less of a difficulty in (ii). Only a few candidates realised that there was a standard series expansion ready to hand for $\ln(\frac{4}{3})$, and those that did generally only went up to the cubed term, which was a shame as the given answer arose from using the next one as well.

The final twist, in part (iv), of giving a range that turned out to be outside the non-zero part of the *pdf*, was twigged by slightly more than half of the candidates that got this far.